Literature Review

EDU8221: Subject Pedagogy in Practice

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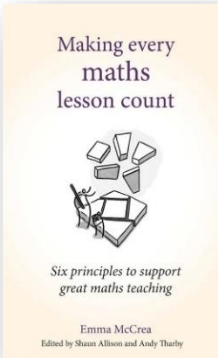
Identify and reflect on what are perceived to be some of the main cognitive challenges within your subject, and the recommended pedagogical approaches to effective teaching and learning to help overcome these challenges.

Context

The placement school wants to advance and nurture students' mathematics skills, with the intent that they can embed the problem-solving skills they obtain through the study of mathematics with the workings of everyday life. At the heart of their philosophy lie the following ideals: to be inclusive, supporting and challenge students positively, irrespective of their background and prior attainment. A final core principle is to implement "the most recent and up to date evidence-informed teaching strategies" (quotation from placement school website) in the classroom, to convey the subject content in a way such that pupils remain engaged and obtain frequent feedback so that they can continue their development independently.

Primary Reading



Clockwise from top-left: (i) *Ofsted, Coordinating Mathematical Success: The Mathematics Subject Report;* (ii) *McCrea, Making Every Maths Lesson Count;* (iii) *EEF, Improving Mathematics in Key Stages 2 and 3: Guidance Report;* (iv) *Kirschner et al, Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experimental and Inquiry-Based Teaching*.

Literature Review

Mathematics is (at least one of) *the* most cognitively demanding subjects taught at secondary school, likely due to the conceptual understanding that it demands of its pupils. In this review, we consider some of the vast literature that studies cognitive challenges in secondary mathematics education and suggest what pedagogical techniques can mitigate these difficulties.

The first challenge we discuss is that of misconceptions. Given the high level of abstraction found across the mathematics curriculum, it should be expected that students may have a higher tendency to misinterpret certain ideas because they are less applicable to the real-world and do not fall within existing schema (upon first learning, at least). Booth and Koedinger (2008, p. 571) look particularly at misconceptions in algebra, their first example being a misunderstanding on "remov[ing] a term from… both sides" of an equation; they identify students' incorrect attempt at solving $x - 4 = 13$ based on them having first understood how to solve $x + 4 = 13$. The underlying issue here seems to be a misconception not solely within algebra, but more fundamentally on the workings of negative numbers. But this grows beyond the abstract. As noted in an exam report (Excel, 2019), students were misunderstanding multiplicative relationships between volumes – note that the discussed exam question had visual aids for the students. Namely, they were mis-claiming that a statement of the form "A is $X\%$ more than B" is equivalent to "B is $X\%$ <u>less</u> than A" (try $X = 100$, for example).

Modelling (particularly in mathematics) is often suggested to mitigate misconceptions, see (Nunes and Bryant, 1996) for example; I agree with this for the most part, and it has been especially useful at assisting pupils in my own practice. This would work nicely with the former example, say by using algebra tiles/bar models for the algebra, or a number line to model the mathematical operation (McCrea, 2019). Although the latter would tackle the issue with the minus, the former is probably the best option from a teaching perspective as it could be applied to other algebraic situations. However, the multiplicative example is interesting in that a pictorial model was provided and yet still the misconception persisted. It is therefore not enough to simply relate the abstract to the tangible. Rather, modelling works better by following a concrete-pictorial-abstract (CPA) approach whereby one scaffolds with concrete objects before allowing them to subside in place of the abstract (McCrea, 2019). This is supported by Fyfe *et al* (2015) in the context of so-called *concreteness fading*; they claim that progressing from concrete to abstract was more fruitful than in the reverse order. In cases where students do not have an existing model to rely on, providing one is good (especially for future-proofing potential further misconceptions) but there are potentially better ways to combat misconceptions in such cases.

One such suggestion I make is for teachers to pose problems and directly question their students. Unlike with modelling, this approach makes clear what a student is doing/wants to do when faced with a problem and allows the teacher an opportunity to understand the process behind the answer and offer feedback accordingly. The EEF (2022) claims feedback as being the most impactful on overcoming misconceptions – they even suggest teachers anticipate misconceptions before they arise. This agrees with our suggestion of using diagnostics. Further to this, one of the concluding remarks of Cavanagh (2007, p. 142) is a suggestion that pupils should be leading conversations with teachers, rather than the converse, so that "misunderstands are more likely to be revealed and more easily rectified". Problem-solving and direct question should therefore be built into classroom practice to root out and address misunderstandings. One key drawback is that the students must be willing to withstand a direct discussion relating to their metacognition. Indeed, Dweck (2016) leads me to suggest that students are likely to disengage with tackling misconceptions if they are afraid of getting things wrong. So we do not suggest an approach tantamount to waiting with bated breath ready to pounce on a misconception. In fact, students may "actively defend their misconceptions" when presented with evidence suggesting they are wrong (Hodgen *et al*, 2018, p. 77). This also ties to mathematical anxiety (we do not discuss this at length, unfortunately), which can greatly diminish so-called *mathematical cognition* as studied by Ashcraft and Ridley (2005). On the contrary, it may be better to *immerse* questioning into the lesson (e.g. in the form of retrieval practice, say). The benefits of operating like this are two-fold: the teacher identifies possible missed misconceptions and, having addressed any such errors, assessing that the misconception has indeed been eradicated. However, it may come at the expense of having less *direct* (one-on-one) interaction since this would interrupt the flow of the lesson. Note that another key ingredient for this to work is for the teacher to be confident in subject (and pedagogical content) knowledge. It feels almost pessimistic to note potential concern here, but the reality is that teachers come from all backgrounds, so some have deeper knowledge than others. An anecdote from (Sherwood, 2022) has the author explain awareness of a misconception but without "ha[ving] solidified why". This is mentioned more generally in the mathematics subject report (Ofsted, 2023, §103), which suggests that misconceptions were inadvertently introduced by teachers will less experience or subject/pedagogical content knowledge. In fact, it is suggested that unguided instruction can foster more misconceptions (Kirschner et al, 2006), so ensuring that the teacher is equipped enough to discuss and navigate around misconceptions is paramount.

In all, it is good that students know what (not) to do, but them being able to understand *why* is a much more important skill; it is this that makes the difference with respect to misconceptions. Modelling can be introduced early as to give pupils a solid grounding of abstract mathematical ideas, but it may not completely circumvent a misconception. Hence, one should use *in tandem* the second suggestion. This way, pupils can both have a pictorial idea of the concepts they encounter, with feedback through direct instruction and questioning to ensure that their model has been correctly embedded.

The second challenge we discuss is language. Mathematics has its own dialect, so it stands to reason that students who struggle with general reading and comprehension can find this subject similarly difficult. In particular, polysemous words are ubiquitous in mathematics and this can be very challenging for some students. In my own practice, I asked students to find a "product" but it was clear many of them did not know the synonymity with multiplication. Although Durkin and Shire (1991) focus on primary-aged children, they suggest students preferred the dominant definition of a polyseme. And unless teachers are actively supporting pupils to overcome such issues, this would ripple up the education system. Language is especially delicate for EAL (Moschkovich, 2002) and SEND pupils (Gersten *et al*, 2008), but some of the strategies we suggest below should help these students also. Parallel to this is, of course, *written* language. It is seemingly rare to have mathematics writing form part of the curriculum explicitly; students most likely learn the written language almost ad hoc. But some educators do encourage good mathematical writing; those that do so the most are themselves more confident in their own knowledge of the subject and pedagogy (Powell *et al*, 2021). The challenge of understanding mathematical language, spoken or written, is partly a demand on cognitive load; students needn't only perform mathematics correctly but be versed well enough to explain what it is they are doing using precise language and setting out their work correctly.

It seems that teachers should exemplify the language they expect their students to use, particularly mathematical vocabulary, and bring explicit attention to polysemes (McCrea, 2019). In doing so, the language will eventually become routine and no longer occupy so much working memory. If teachers use this also as an opportunity to praise pupils for using correct terminology, then the students will see further development in their mathematical communication (Yackel and Cobb, 1996). It is claimed that teachers who favour slang over established nomenclature can instil the same rhetoric in their pupils (Ofsted, 2023). Despite this, I think there is a place in the classroom for colloquialisms; as long as the phrasing doesn't allow for misinterpretation, using idioms – calling a function a *machine* to build a mental model (Pimm, 1987) – can gain intuition, on which mathematical rigour will lay. Furthermore, it suggested by Countryman (1992) that an increase in written fluency also improves reasoning skills. As a means to do this, educators could use formative assessment to check and improve language, in particular the written work (Bryant and Bryant, 2008). If approached informally, say with mini-whiteboards, teachers could really push for correct vocabulary and answer style without the fear of making *permanent* – bookwork – errors. That said, even effective use of formative assessment can lead to student anxiety (Black and William, 1998), so some care has to be taken here.

To finish, Beck *et al* (2013, p. 30) argue that “learning the meaning of the words [in mathematics] is not the point” whereas the underlying concept is. But I have to disagree with this; to truly learn mathematics, pupils have to appreciate both the jargon and the ideas they convey. Moreover, if a student writes an incorrect answer, then they have not produced correct mathematics. ‘Language’ in this sense is slightly reductive: we can convey ideas by speaking, irrespective of the way we speak and how correct we are. But with mathematics, these operate in tandem.

We have considered two (out of *many*) cognitive challenges faced by mathematics pupils. Although the literature provides a smattering of options as to how best combat these difficulties, there are benefits and drawbacks to all. But ultimately, it boils down to the teacher coming well-prepared to a classroom environment that can foster positive learning. Anticipation and preparation are king.

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